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Wave Induced Oscillation in an Irregular Domain by using Hybrid Finite Element Model

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Abstract. Harbors are designed to provide the safe loading, unloading and sheltering for the moored vessels. In seasonal weather conditions, semi-enclosed harbors experienced high resonance, which amplifies the surface wave amplitude in the interior of the harbor due to combined effect of wave refraction, diffraction and partial reflection from the solid harbor walls. An accurate description boundary of the harbor is required to analyze the impact of resonant frequency waves over surface ocean wave height. The fluid domain is divided into two regions as bounded and open sea region. The bounded region consists of harbor boundary and interior of the harbor and open sea region consists of ocean area outside the bounded region. Firstly, the mild slope equation (MSE) is derived for both the regions in terms of a potential function using the energy conservation principle. The total wave energy in the bounded region is estimated by using a mathematical model based on Hybrid Finite Element Method (HFEM) is used to formulate the mild slope equation. In HFEM model, the finite element method is coupled with the boundary element method to solve the mild slope equation in both the region. Further, the present HFEM model is validated with existing studies lead by Lee (1971) and Ippen and Goda (1963). The current numerical model is implemented on realistic Pohang New Harbor (PNH), which is situated on the southeast coast of South Korea. The present numerical model can be used as an efficient engineering tool for planning and designing of the artificial industrial harbor and predict the incident wave response under the resonance conditions.

1. Introduction

Most of the existing harbors connecting with the open sea are experienced the harbor oscillations due to the incident waves propagating from the open sea into the harbor. The small amplitude waves are diffracted, refracted and partially or fully reflected by the exterior and interior boundaries of the harbor walls and a small portion of them are re-radiated through the entrance to the open sea again. The Pohang New harbor (PNH) is built to support the POSCO steel industry in South Korea, which is one of the most powerful steel production plants since the 1970s. Occasionally, the incoming waves to PNH have large amplitudes while they are generated by the seasonal typhoons, winds or currents and swells. Such phenomena are typically observed in Korea coastal region.

Several researchers have carried out significant investigations regarding the prediction of the wave field in a harbor due incident waves of the short period by adopting various numerical schemes. Initially, the mild-slope wave equation was solved by Berkhoff [1] using a finite element technique. Chen and Mei [2] introduced a different version of hybrid element method to solve the shallow water equation. Researchers have developed a different numerical model based on Helmholtz equation and mild slope equation (MSE) [3-8]. These models are very useful to modify or redesign the existing

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harbors. Lee and Williams [9] developed a boundary element numerical model to predict the wave field for the multidirectional random waves in a harbor due to diffraction and refraction. Researchers have designed the time-dependent numerical model with MSE and amplitude dispersion [10]. They transformed extended MSE to the simple time-dependent hyperbolic equation. Tsai and his colleagues [11] have proposed a wave model with both dispersive and nonlinearity for wave field without water depth restriction. They derived linear and nonlinear effects to the second order of MSE for wave steepness and bed slope for narrow-banded sea waves.

In this paper, MSE is utilized to determine the wave response in PNH. Further, the MSE is solved by using the Hybrid Finite Element method for an irregular domain. The kinematic and dynamics free surface boundary condition is utilized to determine the amplification factor in an irregular domain. The consideration of partial wave reflection, diffraction, dissipation losses due to absorption and wave absorption due to breakwaters is incorporated in the present model. Further incident waves with different wave frequencies and directions are also analyzed in PNH ocean surface wave field is estimated for interior and exterior harbor region under the resonance conditions. The safe location in the harbor can be identified under the resonance conditions during the seasonal weather for the safety of moored vessels and ships.

2. Mathematical Formulation

The inner bounded region Ω_b includes the domain D with partially reflecting wall ∂D with outward normal vector n_D and the boundary of the inner bounded region is denoted as $\partial\Omega_b$ shown in Fig. 1. The exterior region outside from bounded region (Ω_b) is defined as an unbounded region (Ω_u). The incident wave propagating from unbounded region towards inner bounded region are partially reflecting and diffracting from the solid boundary ∂D. Thus, partial reflection boundary is given by Isaacson and Qu [12]

$$\frac{\partial \phi_b}{\partial n_D} + ik\alpha_p \phi_b = 0 \tag{1}$$

where n_D is outward normal vector and $\alpha_p = \alpha_p^1 + i\alpha_p^2$ is transmission coefficient. It is defined in terms of reflection coefficient K_r, which can be written as the ratio of a reflected wave height to an incident wave height. Here, the transmission coefficient is considered as $\alpha_p^1 = 0$ and $\alpha_p^2 = 1 - K_r / 1 + K_r$. The velocity potential satisfied the mild slope equation in the inner bounded region Ω_b defined as

$$\nabla \cdot (CC_g \nabla \phi_b) + \omega^2 \frac{C_g}{C} \phi_b = 0 \tag{2}$$

where $\phi_b = \phi_b(x, y)$ is the potential function in Ω_b, ω is angular frequency, $C(x, y)$ is phase velocity, $C_g(x, y)$ is the group velocity, i.e., $C_g(x, y) = nC$ such that $n = \left(1 + 2kh / \sinh kh\right) / 2$, where k is the wave number related to water depth through dispersion relation as $\omega^2 = gk \tanh kh$. The region exterior to the boundary $\partial\Omega_b$ is denoted as an unbounded region Ω_u. Incident wave denoted as an arrow toward to bounded region. The Somerfield radiation boundary condition is given on fictitious boundary $\partial\Omega_b$. The matching boundary conditions is given over the boundary $\partial\Omega_b$, the velocity potential for unbounded region ϕ_u and bounded region ϕ_b are equal, i.e.,

$$\phi_b = \phi_u \quad \text{and} \quad \frac{\partial \phi_b}{\partial n_b} = \frac{\partial \phi_u}{\partial n_u} \tag{3}$$

In unbounded region, the velocity potential can be defined as

$$\phi_u = \phi_{inc} + \phi_{scr} \tag{4}$$

where ϕ_{inc} represent the incident wave potential and ϕ_{scr} is scattering wave potential. In the unbounded region, radiated waves propagate to far field from the center of Ω_u. Thus, the Somerfield radiation boundary condition must satisfy, i.e., $lim_{r\to\infty}\sqrt{r}\left(\partial/\partial r-ik\right)\phi_{scr}=0$, where r is the radius from the center of Ω_b to the fictitious far field boundary $\partial\Omega_b$ in unbounded region.

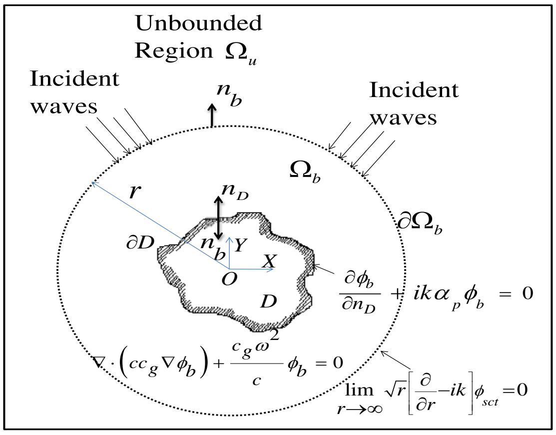


Fig. 1. In the bounded region Ω_b includes the domain D with curved solid boundary ∂D and outward normal vector n_D. A closed circular arc $\partial\Omega_b$ with outward normal vector n_b is also given.

Energy flux in the bounded region can be defined as

$$E_{f\,\Omega_b}=\frac{\rho}{g}\left[\left\{\frac{1}{s}\iint_{\Omega_b}\left(CC_g\left(\bar{\nabla}\phi_b\right)^2-\frac{C_g}{C}\phi_b\right)dxdy\right\}-\left\{\int_{\partial\Omega_b}\frac{1}{2}CC_g\phi_b\frac{\partial\phi_b}{\partial n_b}ds-\int_{\partial D}\frac{1}{2}i\alpha_p\omega C_g\phi_b^2ds\right\}\right]e^{-2i\omega t} \tag{5}$$

Here bar represents the complex conjugate. Moreover, the Energy flux acting in the unbounded region and along the boundary $\partial\Omega_b$ is given as

$$E_{f\,\Omega_u}=\frac{\rho}{g}\left[\frac{1}{2}\left\{\int_{\partial\Omega_b}CC_g\phi_u\frac{\partial\phi_u}{\partial n_u}ds-\int_{\partial\Omega_b}CC_g\phi_{scr}\frac{\partial\phi_{scr}}{\partial n_u}ds\right\}-\left\{\int_{\partial\Omega_b}\frac{1}{2}CC_g\phi_{inc}\frac{\partial\phi_{inc}}{\partial n_u}ds-\int_{\partial\Omega_b}CC_g\phi_{scr}\frac{\partial\phi_{inc}}{\partial n_u}ds\right\}\right]e^{-2i\omega t} \tag{6}$$

From conservation of energy, i.e., $\partial E_{\Omega_b}/\partial t=0$ and $\partial E_{\Omega_u}/\partial t=0$, therefore E_{Ω_b} and E_{Ω_u} must be constant. The matching boundary conditions Eq. (3) and Eq. (4) are utilized to formulate Eq. (5) and Eq. (6).

3. Numerical validation and discretization

In this experiment, the rectangular harbor is used as the same size which is used by Ippen and Goda (1963) in their experiment. The triangulation mesh is generated and the rectangular domain is discretized into 12564 elements.

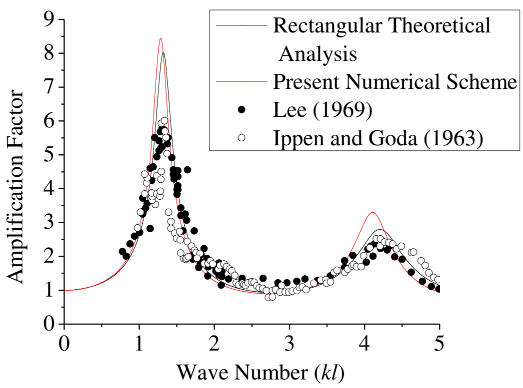
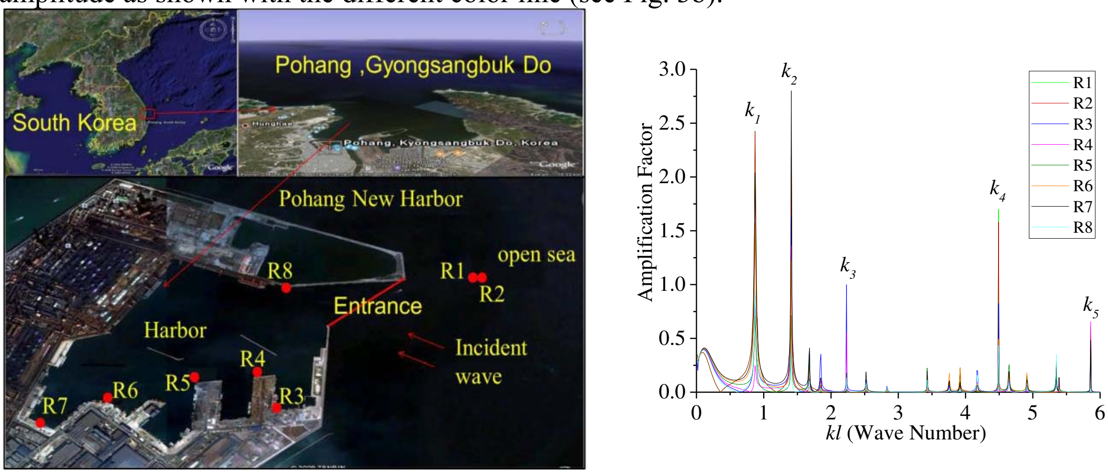


Fig. 2. Comparison of the present scheme (red line) with the experimental results (by Lee and rectngular harbor theory

As shown in Fig. 2, the hollow circle shows the experimental result of Ippen and Goda [13], a solid circle denotes the experimental result of Lee [14], the red line shows numerical data of present scheme, the black line shows the data of rectangular theory. The present numerical simulation results of HFEM model have validated with the rectangular harbor theoretical results and existing experimental results. Therefore, HFEM model can be applied to realistic harbor domain such as Pohang New Harbor (PNH).

4. Implementation on PNH

The present HFEM model is applied to realistic Pohang New Harbor (PNH). Recorder stations R3 to R8 are chosen as the interior of PNH boundary especially at locations of moored ships, and R1 and R2 are chosen as reference locations exterior of PNH (see Fig. 3a). After confirmation of validation of present numerical scheme, the HFEM model is applied to PNH for estimating the amplification factor at various recorder stations from R1 to R8. For each recorder station, the amplification factor varied in amplitude as shown with the different color line (see Fig. 3b).



(a) (b)

Fig. 3. The topographic view of PNH with recorder stations is given as R1 to R8 in Pohang New Harbor, including open sea region and incident wave (denoted by a red arrow). The amplification factor at different recorder stations from R1 to R8 is compared with respect to wave number (kl), where l is the length of the entrance of PNH.

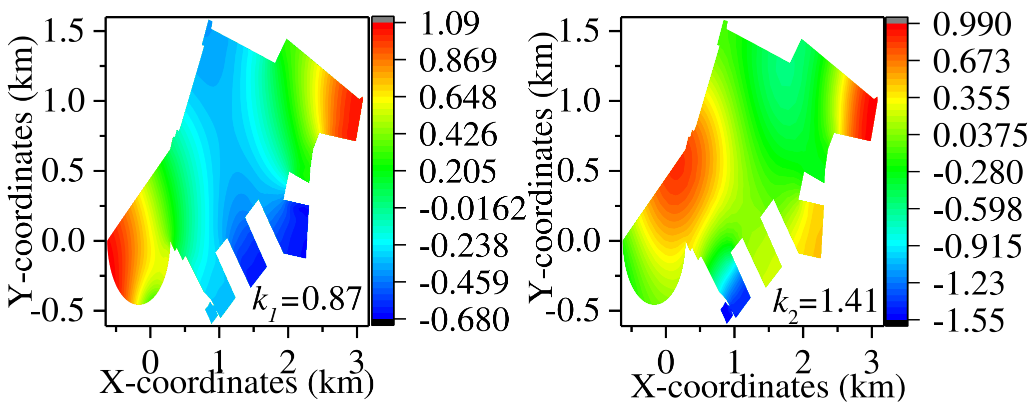


Fig. 4. Wave field analysis for the PNH at first and second resonance mode K_1=0.87, K_2=1.41.

The wave field analysis for the PNH is also conducted for the incident waves propagating at various incident angles. In Fig. 4, the ocean surface wave field is given for the first and second resonance modes with an incident angle $\alpha = 3\pi/4$ at the entrance. There are two major peaks appears in the PNH for first resonance mode and three major peaks for second resonance mode. Similarly, we can obtain the surface wave elevation in the PNH for different resonance modes and incident angles. On the basis of wave field information , the safe location for the moored ship can be identified inside the PNH during the seasonal weather conditions.

5. Discussion and conclusion

In this paper, the mild slope equation (MSE) is utilized to determine the wave potential function and MSE is solved using HFEM. In this formulation, partial reflection boundary at the harbor and Somerfield radiation boundary condition are applied for far field. The comparison of the amplification factor in the rectangular harbor for a present model with analytical approximation and experimental results of existing studies are given. Physical models are very expensive and time-consuming; however numerical model proposed here is effective and sufficiently accurate to analyze the resonance phenomenon in the harbor complex geometry. Therefore, we can utilize such models as a reliable engineering tool to predict the wave frequencies and amplitude in the harbor. Further, the moored ship can be protected from the high amplitude waves during seasonal weather conditions.

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